TIME INHOMOGENEOUS COUPLINGS OF DIFFUSION PROCESSES ON RIEMANNIAN MANIFOLDS

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ABSTRACT. In this talk, I will exhibit some examples of coupling methods for diffusion processes of different speed on (Riemannian) manifolds. It typically appears on spaces with a time-dependent metric, such as (backward) Ricci flow. In this case, we can recover a monotonicity of some transportation costs which naturally arises in the geometry of Ricci flow. Even on a Riemannian manifold with time-independent metric, we can obtain an estimate of L^2-transportation cost between distributions of diffusion processes at different times by using a time-inhomogeneous coupling method. It is closely related with Bakry-Ledoux's gradient estimate for diffusion semigroups or Bochner's inequality, which are analytic formulations of the curvature-dimension condition. Our construction of couplings relies on an approximation of diffusion processes by geodesic random walks. It works well, especially in time-inhomogeneous setting, to avoid technical difficulties coming from the presence of cut-locus.