ON THE PRECISE GAUSSIAN HEAT KERNEL LOWER BOUNDS

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ABSTRACT. Assume that the volume doubling condition and the scaled Poincare inequalities hold for a strongly local regular Dirichlet form on a metric measure space. Then, it is known that there exists a jointly continuous heat kernel that satisfies both side Gaussian estimates. We are interested in the following sharper estimates: For each $\epsilon > 0$, there exists $c_\epsilon > 0$ such that

$$(c_\epsilon \mu(B(x,t^{1/2})))^{-1} \exp(-\frac{d(x,y)^2}{(4-\epsilon)t}) \le p_t(x,y) \le c_\epsilon(\mu(B(x,t^{1/2})))^{-1} \exp(-\frac{d(x,y)^2}{(4+\epsilon)t})$$

for all x, y and all $0 < t < 1$. By the well-known method of Davies, the upper bound holds in general, so the question is the lower bound. Varopoulos (1990) obtained such estimates for sub-Laplacians on Nilpotent groups and for divergence forms on C^∞ manifolds assuming suitable uniform elliptic conditions. In this talk we will give conditions that imply the precise lower bound. We can verify the conditions for some concrete examples. Especially, for the case of divergence forms on C^∞ manifolds, we can weaken the smoothness condition of the coefficients of the form considerably. This talk is based on the joint work with L. Saloff-Coste (Cornell).